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ABSTRACT

By means of a certain transformation, a relationship is demonstrated between a class of two-dimensional and three-dimensional scalar or electromagnetic diffraction problems. The basic three-dimensional configuration consists of a perfectly reflecting half plane excited by a ring source centered about the edge and having a variation $\exp(\pm i\phi/2)$, where ϕ is the azimuthal variable; in addition, a perfectly reflecting rotationally symmetric obstacle whose surface is defined by $f(\rho, z) = 0$ (ρ, z are cylindrical coordinates), may be superposed about the edge (z-axis). This problem is shown to be simply related to the two-dimensional one for the line source excited configuration $f(y, z) = 0$, where y and z are Cartesian coordinates. Various special obstacle configurations are treated in detail.

For the general case of arbitrary electromagnetic excitation, the above-mentioned transformation is used to construct the solution for the diffraction by a perfectly conducting half plane from the knowledge of appropriate scalar solutions, namely those which obey the same equations and boundary conditions, and have the same excitations, as the Cartesian components of the electromagnetic field.

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I. Introduction

This paper deals with the relationship between a special class of two-dimensional and three-dimensional diffraction problems. The basic three-dimensional configuration consists of a perfectly reflecting half plane excited by a ring source centered about the edge, with the plane of the loop oriented perpendicular to the edge, as shown in Fig. 1(a); the strength of the source, either scalar or electromagnetic, varies like $\sin(\phi/2)$ or $\cos(\phi/2)$, where ϕ is the azimuthal angular variable. Via the transformation

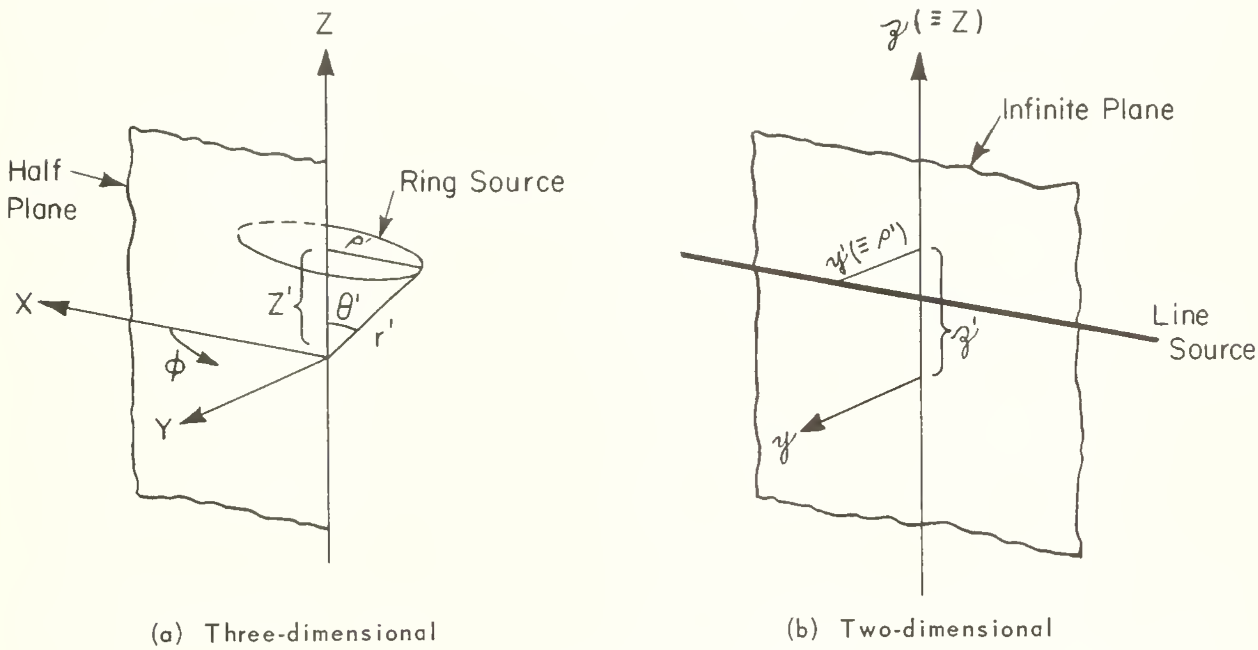
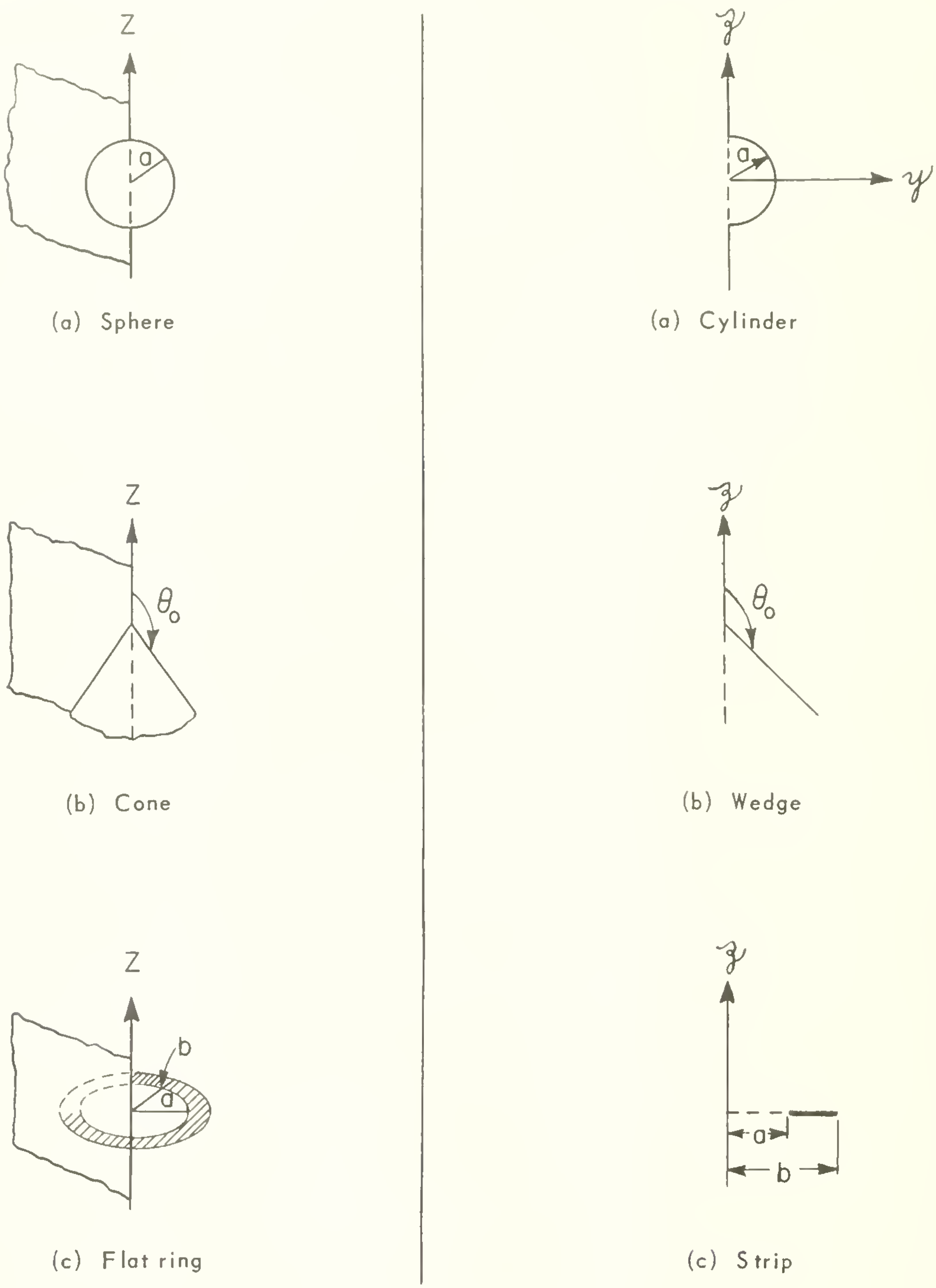


Fig. 1 - Basic equivalent configurations

to be described below, the cylindrical coordinate $\rho = r \sin\theta$ in Fig. 1(a) transforms into the y-coordinate in Fig. 1(b), while the Z-coordinate is preserved (the x-coordinate does not occur in the two-dimensional problem). The three-dimensional configuration in Fig. 1(a) is thereby transformed into the two-dimensional one in Fig. 1(b) consisting of a uniform line source parallel

Three-dimensional | Two-dimensional



(a) Sphere — (a) Cylinder

(b) Cone — (b) Wedge

(c) Flat ring — (c) Strip

Fig. 2 - Equivalent configurations

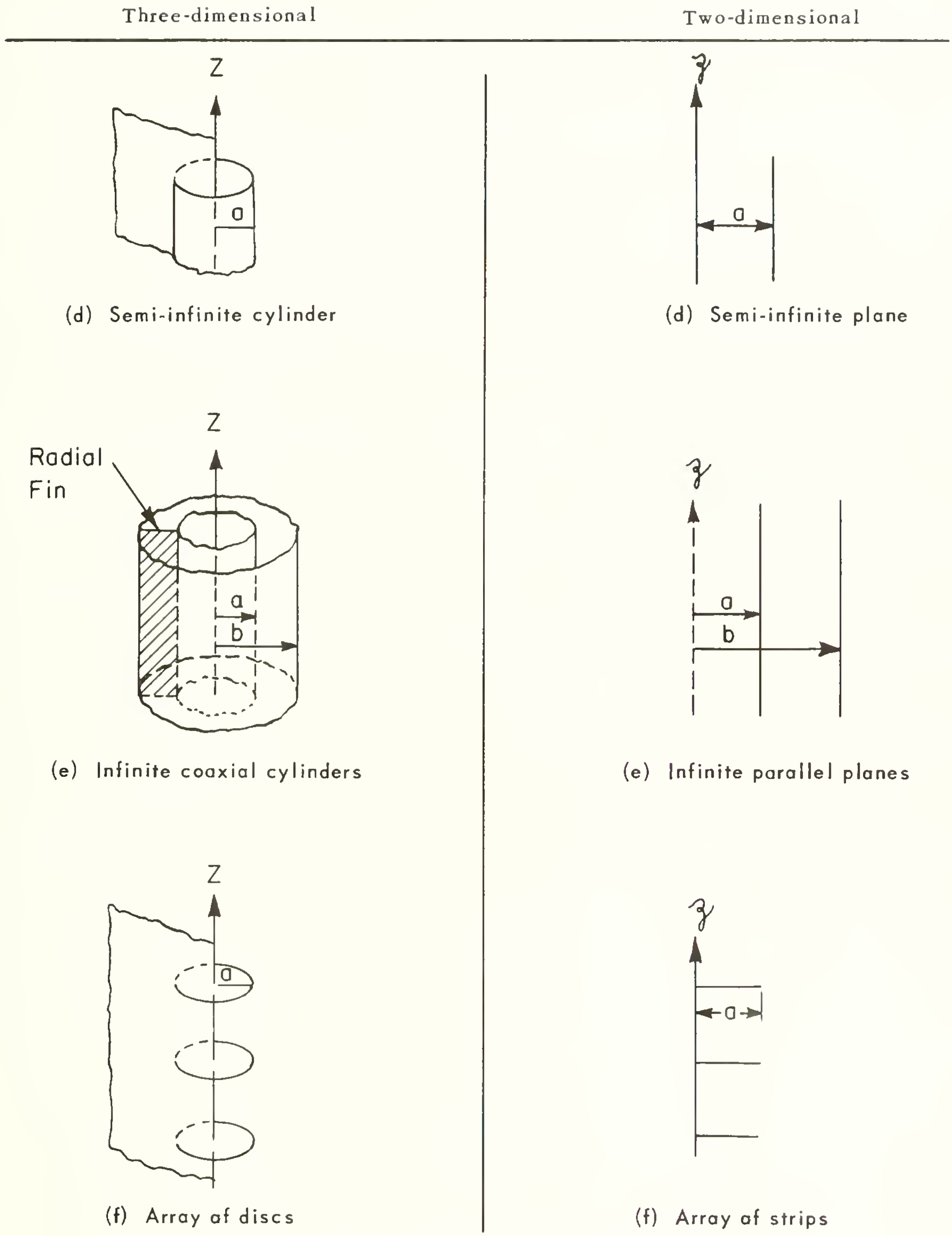


Fig. 2 - Equivalent configurations.

to a perfectly reflecting infinite plane. It is evident that any surface with rotational symmetry about the Z-axis of Fig. 1(a), and described by the equation $f(\rho, Z) = 0$, is mapped by this transformation into the two-dimensional configuration $f(y, Z) = 0$ in the presence of the infinite plane at $y = 0$. Some special structures in this category are listed in Fig. 2. We shall show how we can generate solutions for such ring source excited three-dimensional configurations involving an infinite half plane, from the knowledge of the two-dimensional results.

As a further application it will be shown how the above-mentioned transformation can be employed to construct the electromagnetic field due to an arbitrary source distribution in the presence of a perfectly conducting half-plane. The construction is carried out explicitly in terms of the solutions of the corresponding scalar Dirichlet and Neumann problems. The problem has been solved previously by Heins[1], Senior[2], and Vandakurov[3], for various dipole excitations through the use of methods which differ from each other and from the present one. The procedure we employ exhibits explicitly the modifications necessary in order to convert the scalar solutions into vector solutions. Thus, we start with the scalar wave functions corresponding to excitations which are Cartesian components of the arbitrarily prescribed vector excitation and which are assumed to be known. The vector solution is then constructed from these scalar solutions.

II. Relation Between Three-and Two-dimensional Problems

A. Scalar Problems

We define the even and odd Green's functions $G_e(\underline{r}; r', \theta')$ and

$G_o(\underline{r}; r', \theta')$ appropriate to the ring source excited half-plane in Fig. 1(a) as

$$G_{\substack{e\\o}}(\underline{r}; r', \theta') = r'\sin\theta' \int_0^{2\pi} \begin{Bmatrix} \cos(\phi'/2) \\ \sin(\phi'/2) \end{Bmatrix} \mathcal{G}_{\substack{e\\o}}(\underline{r}, \underline{r}')d\phi', \quad \underline{r} = (r, \theta, \phi), \tag{1}$$

where $\mathcal{G}_e(\underline{r}, \underline{r}')$ and $\mathcal{G}_o(\underline{r}, \underline{r}')$ are the three-dimensional Green's functions satisfying the inhomogeneous wave equation

$$\left(\nabla^2_{r\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2} + k^2\right)\mathcal{G}_{\substack{e\\o}}(\underline{r}, \underline{r}') = -\frac{\delta(r-r')\,\delta(\theta-\theta')\,\delta(\phi-\phi')}{r'^2\sin\theta'} \tag{2}$$

$$\nabla^2_{r\theta} = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}\ , \tag{2a}$$

in the domain $0 \le r < \infty$, $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, with the boundary conditions on the half-plane

$$\frac{\partial\mathcal{G}_e}{\partial\phi} = 0, \quad \mathcal{G}_o = 0, \quad \text{at} \quad \phi = 0,\ 2\pi\ , \tag{3a}$$

the edge condition[4]

$$\mathcal{G}_{\substack{e\\o}} \text{ finite at } \rho = r\sin\theta \to 0\ , \tag{3b}$$

and also the

$$\text{radiation condition at } r \to \infty\ . \tag{3c}$$

One notes from (1) that G_e and G_o are excited by ring sources with $\cos(\phi/2)$ and $\sin(\phi/2)$ variation, respectively. Since these sources generate fields which vary everywhere like $\cos(\phi/2)$ and $\sin(\phi/2)$, respectively, we may represent $G_{\substack{e\\o}}$ as

$$G_{\substack{e\\o}}(\underline{r}; r', \theta') = \begin{Bmatrix} \cos(\phi/2) \\ \sin(\phi/2) \end{Bmatrix} G(\underline{\rho}, \underline{\rho}'), \quad \underline{\rho} = (r, \theta)\ , \tag{4}$$

whence G satisfies the following equation in view of (1), (2) and (4):

$$\left(\nabla^2_{r\theta} - \frac{1}{4r^2\sin^2\theta} + k^2\right) G(\underline{\rho},\underline{\rho}') = -\frac{\delta(r-r')\,\delta(\theta-\theta')}{r'} \qquad (5)$$

with boundary conditions as in (3b) and (3c).

We now introduce the transformation

$$G(\underline{\rho},\underline{\rho}') = \sqrt{\frac{\rho'}{\rho}}\,\overline{G}(\underline{\rho},\underline{\rho}') \;, \quad \rho = r\sin\theta \;, \; 0 \leq \rho < \infty \;, \qquad (6)$$

into (5), and find that $\overline{G}$ satisfies

$$\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + k^2\right) \overline{G}(\underline{\rho},\underline{\rho}') = -\frac{\delta(r-r\;)\,\delta(\theta-\theta')}{r'} \quad (7a)$$

If r and θ in (7a) are interpreted as cylindrical coordinates in a Cartesian (y,z) space where

$$\begin{aligned} y &= r\sin\theta \;, \quad 0 \leq y < \infty \;, \\ z &= r\cos\theta \;, \quad -\infty < z < \infty \;, \end{aligned} \qquad (7b)$$

then one notes that (7a), together with the boundary conditions in (8) below, represents exactly the formulation for the two-dimensional Green's function problem in Fig. 1(b). This establishes the desired relationship between the ring-source-excited three-dimensional problem and the line-source-excited two-dimensional problem. To avoid confusion between the spherical and cylindrical coordinate interpretations of r, θ in (5) and 7(a), respectively, we shall henceforth employ the rectangular (y,z) coordinate notation for the equivalent two dimensional problem (see Fig. 1(b)). Concerning the boundary conditions on $\overline{G}$ in (7a), we impose the radiation condition at infinity, as before, and on the infinite plane at y = 0, we require in view of (3b), (6), and (7b),

$$\overline{G} = 0 \quad \text{at} \quad y = 0 \qquad (8)$$

Since this implies that $\overline{G} \propto y$ as $y \to 0$, it follows that $G \propto \sqrt{\rho}$ as $\rho \to 0$, in conformity with the familiar edge requirement.[4]

The basic transformation employed above can also be expressed as the following theorem and its converse which are verifiable by direct calculation.

<u>Theorem</u>: If

$$X = r \sin \theta \cos \Phi;\ Y = r \sin \theta \sin \Phi;\ Z = r \cos \theta; \qquad (9)$$

$$\rho \equiv y = r \sin \theta; \quad Z \equiv z = r \cos \theta,$$

$$\frac{\partial^2 \overline{u}}{\partial \rho^2} + \frac{\partial^2 \overline{u}}{\partial z^2} + k^2 \overline{u} = 0; \quad \overline{u} = \overline{u}(\rho, z) \equiv \overline{u}(y, z) \ ,$$

$$U(X,Y,Z) = \frac{1}{\sqrt{\rho}} \ \exp(\pm\ i\ \Phi/2)\ \overline{u}(\rho, z) \ ,$$

then

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} + k^2\right) U(X,Y,Z) = 0 \ . \qquad (9a)$$

Conversely, if (9a) is true, and $U(X,Y,Z) = \frac{1}{\sqrt{\rho}} \exp(\pm\ i\Phi/2)\ \overline{u}(\rho,z)$ then $\overline{u}_{\rho\rho} + \overline{u}_{zz} + k^2\overline{u} = 0$.

If the configuration also includes a surface of revolution S defined by the equation $f(\rho, z) = 0$, all previous considerations apply except that, in addition, the boundary conditions on S must be taken into account. Since the surface S is independent of the Φ-coordinate, the boundary conditions need be imposed only on $\overline{G}(y,z;y',z')$ along the curve $f(y,z) = 0$. Let us assume the linear homogeneous boundary condition

$$G = \alpha \frac{\partial G}{\partial n} \text{ on S} \ , \quad \alpha = \text{constant} \ , \qquad (10)$$

where n is the direction of the normal into S. Then the condition on $\overline{G}$ is found via (6) as

$$\overline{G} = \alpha \left[\frac{\partial}{\partial \overline{n}} - \frac{1}{2y}\left(\frac{\partial y}{\partial \overline{n}}\right)\right] \overline{G} \quad \text{on} \quad \overline{S} \ , \tag{11}$$

where $\overline{S}$ is the surface $f(y,z) = 0$ and $\overline{n}$ the direction of the normal into $\overline{S}$. One notes from (10) and (11) that for $\alpha = 0$

$$\overline{G} = 0 \quad \text{on} \quad \overline{S} \quad \text{if} \quad G = 0 \quad \text{on} \quad S \ , \tag{12a}$$

while for $\alpha = \infty$,

$$\frac{\partial \overline{G}}{\partial \overline{n}} = \frac{1}{2y}\left(\frac{\partial y}{\partial \overline{n}}\right) \overline{G} \quad \text{on} \quad \overline{S} \quad \text{if} \quad \frac{\partial G}{\partial n} = 0 \quad \text{on} \quad S. \tag{12b}$$

Thus, a Dirichlet condition ($\alpha = 0$) on S always implies the same condition on $\overline{S}$, whereas a Neumann condition ($\alpha = \infty$) on S leads generally to a mixed boundary condition on $\overline{S}$, unless the obstacles are confined completely to the planes z = constant, in which case $(\partial y/\partial \overline{n}) = \pm (\partial y/\partial z) = 0$.

We have therefore shown how the solutions for any two-dimensional Dirichlet type diffraction problem can be taken over to yield the solution for a corresponding three-dimensional Dirichlet problem with an azimuthal field variation as in (4). Typical examples for which exact two-dimensional solutions are known include the cylinder (Fig. 2(a)), the wedge (Fig. 2(b)),* the slit (Fig. 2(c), with $b \rightarrow \infty$), the semi-infinite parallel plane region (Fig. 2(d)) and the infinite parallel plane region (Fig. 2(e)). Approximate solutions are available for the infinite array of strips (Fig. 2(f)), and others. The corresponding three-dimensional Dirichlet problems solved

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* A detailed discussion of the relationship between the two-dimensional wedge problem and the corresponding three-dimensional cone problem is given in a report by Felsen.[5]

thereby via (4) and (6) are also shown in Fig. 2. Concerning the Neumann type boundary condition, solutions for the two-dimensional problems in Figs. 2(c) and 2(f) imply those for the corresponding three-dimensional case, as noted above. Moreover, for the configurations in Figs. 2(d) and 2(e), a Neumann condition on the cylindrical surfaces implies an impedance type boundary condition on the corresponding two-dimensional plane surface since on a plane y = constant, $(1/y)\,(\partial y/\partial \bar{n})$ is equal to a constant. This solution is known for the infinite parallel plane case. Similarly, the Neumann condition $\partial G/\partial r = 0$ at $r = a$ appropriate to a sphere of radius "a" has for its two-dimensional equivalent the known solution for a cylinder with $(\partial \bar{G}/\partial \bar{n}) = (1/2a)\bar{G}$, while that for a cone, $\partial G/\partial \theta = 0$ at $\theta = \theta_o$, leads[6] to a simple two-dimensional wedge problem with $\partial \bar{G}/\partial \theta = (1/2)\,(\cot\theta_o)\bar{G}$.

B. Electromagnetic Problems

Now we show how the fields radiated in the presence of a perfectly conducting half-plane by electromagnetic ring source distributions with variations $\cos(\phi/2)$ or $\sin(\phi/2)$ can be inferred from the solution for a scalar line source in the presence of an infinite plane. The latter solution can, of course, be expressed simply in terms of the free space result plus its image. Several other boundary value problems are also discussed.

1. Longitudinal Electric Currents

The configuration is shown in Fig. 3. A source variation $\sin(\phi/2)$ is assumed since a tangential electric source current element of finite strength cannot exist on a perfectly conducting sheet. The vector electric field $\underline{\mathcal{E}}(\underline{r};r',\theta')$ and magnetic field $\underline{\mathcal{H}}(\underline{r};r',\theta')$ radiated by this distribution of currents of unit strength are given in terms of the Hertzian vector $\underline{z}_o\, G_o$

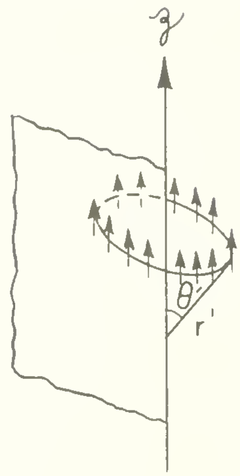

Fig. 3 - Ring of longitudinal current elements

by

$$\underline{\mathcal{E}}(\underline{r};r',\theta') = -\frac{\mathcal{Z}}{ik}\nabla\times\nabla\times\left[\underline{z}_o\, G_o(\underline{r};r',\theta')\right],\ \underline{\mathcal{H}}(\underline{r};r',\theta') = \nabla\times\left[\underline{z}_o\, G_o(\underline{r};r',\theta')\right], \tag{13}$$

where $\mathcal{Z}$ is the characteristic impedance of free space and $\underline{z}_o$ a unit vector along the z-direction. A time dependence $\exp(-i\omega t)$ is implied. Since G_o as given in (4) can be related to the two-dimensional problem in Fig. 1(b) via (6), (7a) and (8) we can write the solution as:

$$\underline{\mathcal{E}}(\underline{r};r',\theta') = -\frac{\mathcal{Z}}{ik}\sqrt{\rho'}\,\nabla\times\nabla\times\left[\underline{z}_o\frac{\sin(\phi/2)}{\sqrt{\rho}}\,\overline{G}(\underline{\rho},\underline{\rho}')\right],\quad \rho = r\sin\theta\ , \tag{14}$$

$$\overline{G}(\underline{\rho},\underline{\rho}') = \frac{i}{4}\left[H_o^{(1)}\left(k\sqrt{(\rho-\rho')^2+(z-z')^2}\right) - H_o^{(1)}\left(k\sqrt{(\rho+\rho')^2+(z-z')^2}\right)\right], \tag{14a}$$

and similarly for $\underline{\mathcal{H}}$. $H_o^{(1)}(w)$ in (14a) is the Hankel function of the first kind of order zero and argument w.

The electric field components along the ρ, ϕ, z directions are given from (13) by

$$\mathcal{E}_\rho = -\frac{\mathcal{Z}}{ik}\frac{\partial^2 G_o}{\partial z\,\partial\rho}, \quad \mathcal{E}_\phi = -\frac{\mathcal{Z}}{ik\rho}\frac{\partial^2 G_o}{\partial z\,\partial\phi}, \quad \mathcal{E}_z = -\frac{\mathcal{Z}}{ik}\left(\frac{\partial^2}{\partial z^2} + k^2\right) G_o , \tag{15}$$

so that one verifies that $\mathcal{E}_z$ and $\mathcal{E}_\rho$ vanish on the half plane as required. Since $\overline{G} \propto \rho$ as $\rho \to 0$ one obtains the proper edge behavior[4]

$$\mathcal{E}_\rho \propto \rho^{-1/2} \sin(\phi/2), \mathcal{E}_\phi \propto \rho^{-1/2} \cos(\phi/2), \mathcal{E}_z \propto \rho^{1/2} \sin(\phi/2) , \tag{16}$$

and analogously for $\mathcal{H}_\rho$ and $\mathcal{H}_\phi$ (note: $\mathcal{H}_z \equiv 0$ from (13)).

It is noted from (15) that if G_o vanishes on the cylindrical surfaces $\rho = a, b$, then $\mathcal{E}_\phi$ and $\mathcal{E}_z$ vanish likewise. Thus, by inserting for $\overline{G}$ in (14) the Dirichlet parallel plane Green's function (see Fig. 2(e)), we can construct the electromagnetic solution for the ring source excited perfectly conducting coaxial waveguide region whose center conductor is supported by a radial fin. The limiting cases $a \to 0$ or $b \to \infty$ are also admitted. By employing the parallel plane eigensolutions we may also determine those E mode functions in the coaxial waveguide whose $\mathcal{E}_z$ component has an azimuthal variation $\sin(\phi/2)$ (only the E modes are excited by present source distribution since $\mathcal{H}_z \equiv 0$ as noted above); the cutoff wavelengths for these modes are identical with those for E modes in the parallel plane waveguide.

2. Longitudinal Magnetic Currents

The physical configuration is as in Fig. 3, where the source distribution is now taken as a ring of unit strength magnetic currents with variation $\cos(\phi/2)$. From considerations dual to those in the preceding section, we

write down the magnetic vector field $\underline{\mathcal{H}}$ for this case as

$$\underline{\mathcal{H}}(\underline{r};\, r',\, \theta') = -\frac{\mathcal{Y}}{ik}\sqrt{\rho'}\,\nabla\times\nabla\times\left[\underline{z}_0\,\frac{\cos(\phi/2)}{\sqrt{\rho}}\,\overline{G}(\underline{\rho},\, \underline{\rho}')\right],\ \mathcal{Y} = \frac{1}{\mathcal{Z}}, \qquad (17)$$

where $\overline{G}$ is given in (14a). The corresponding expression for the electric field $\underline{\mathcal{E}}$ has a form dual to that given for $\underline{\mathcal{H}}$ in (13).

For the perfectly conducting waveguide with the radial fin (Fig. 2(e)), it is required that $\partial G_e/\partial\rho = 0$ at $\rho = a$, b (cf. (15), with $\mathcal{E}$, $\mathcal{Z}$, G_0 replaced by $\mathcal{H}$, $\mathcal{Y}$, G_e, respectively). The corresponding parallel plane problem therefore is one for which the impedance boundary condition (cf. (12b))

$$\frac{\partial\overline{G}}{\partial y} = \mp\frac{1}{2y}\,\overline{G} \text{ at } y = {a \atop b} \qquad (18)$$

applies. By employing the parallel plane eigensolutions satisfying (18), we can construct via (17) those H mode functions whose $\mathcal{H}_z$ field component varies like $\cos(\phi/2)$ in the coaxial waveguide with the radial fin. Again, the cut-off wavelengths for these modes in the coaxial guide are identical with those for the H modes in the parallel plate guide with the impedance type boundary conditions (18).

3. Radial Electric Currents

The physical configuration for a ring of electric currents of unit strength directed radially with respect to some arbitrarily chosen origin along the edge of the half plane and varying like $\sin(\phi/2)$ is shown in Fig. 4. The electric and magnetic fields due to this current distribution are given in terms of a radial Debye potential function $\underline{r}\, G_0$ by

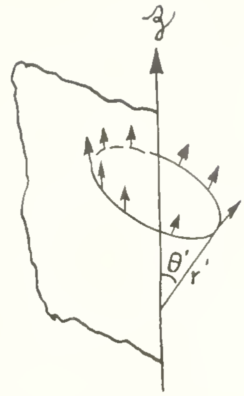

Fig. 4 - Ring of radial current elements.

$$\underline{\mathcal{E}}(\underline{r};r',\theta') = -\frac{Z}{ik}\ \nabla\times\nabla\times\left[\frac{\underline{r}}{r'}\,G_o(\underline{r};r',\theta')\right],\ \underline{\mathcal{H}}(\underline{r};r',\theta') = \nabla\times\left[\frac{\underline{r}}{r'}\,G_o(\underline{r};r',\theta')\right] \tag{19}$$

where $\underline{r} = \underline{r}_o\, r$, $\underline{r}_o$ is the radial unit vector, and G_o is related to $\overline{G}$ in (14a) via (4) and (6) by $G_o = \sqrt{\rho'/\rho}\,\sin(\phi/2)\,\overline{G}(\underline{\rho},\underline{\rho}')$.

One finds from (19) that the r, θ, ϕ - components of the electric field depend on G_o in (4) in the following manner:

$$\mathcal{E}_r \propto \left(\frac{1}{r^2}\frac{\partial}{\partial r}\,r^2\,\frac{\partial}{\partial r} + k^2\right) G_o\ ,\quad \mathcal{E}_\theta \propto \frac{1}{r}\,\frac{\partial^2}{\partial r\,\partial\theta}\ G_o,\ \mathcal{E}_\phi \propto \frac{1}{r\sin\theta}\ \frac{\partial^2}{\partial r\,\partial\phi}\ G_o\ . \tag{20}$$

It is verified readily that the edge conditions (16) are satisfied as $\rho \to 0$. Furthermore, one notes that if G_o, and therefore $\overline{G}$, vanishes on the conical boundary θ = constant, then $\mathcal{E}_r$ and $\mathcal{E}_\phi$ likewise vanish. Thus, (19) remains valid when a perfectly conducting cone is superposed on the edge, with apex at the origin, as in Fig. 2(b), provided that one inserts for $\overline{G}$ the corres-

ponding two-dimensional Dirichlet solution for the wedge.

If a perfectly conducting sphere with radius "a" and centered at the origin is superposed onto the half-plane (Fig. 2(a)), one notes from (20) that the required boundary conditions are met if $\partial G/\partial r = 0$ at $r = a$. The corresponding boundary condition on the cylinder in the equivalent two-dimensional problem is given as before by $\partial \bar{G}/\partial \bar{n} = (1/2a)\, \bar{G}$, leading again to a known boundary value problem.

4. Radial Magnetic Currents

The magnetic field radiated by a ring source of unit strength radial magnetic current elements as in Fig. 4 is obtained from an expression dual to that in (19) (a $\cos(\phi/2)$ source variation is assumed):

$$\underline{\mathcal{H}}(\underline{r};r',\theta') = -\frac{\mathcal{Y}\sqrt{\rho'}}{ikr'}\,\nabla\times\nabla\times\left[\underline{r}\,\frac{\cos(\phi/2)}{\sqrt{\rho}}\,\bar{G}(\underline{\rho},\underline{\rho}')\right] \tag{21}$$

with $\bar{G}$ given in (14a). The corresponding expression for $\underline{\mathcal{E}}$ has a form dual to that for $\underline{\mathcal{H}}$ in (19).

If a perfectly conducting sphere with radius "a" is centered at the origin as in Fig. 2(a), the required boundary conditions on $\underline{\mathcal{H}}$ in (21) are satisfied if $G = 0$ at $r = a$, leading to the equivalent two-dimensional Dirichlet problem for the cylinder. For a perfectly conducting conical boundary at $\theta = \theta_o$ (Fig. 2(b)), the required boundary condition is $\partial G/\partial\theta = 0$ at $\theta = \theta_o$, leading via (12b) to the previously mentioned equivalent two-dimensional wedge problem with $\partial\bar{G}/\partial\theta = (1/2)\,(\cot\,\theta_o)\,\bar{G}$.

III. Diffraction of Electromagnetic Fields by a Perfectly Conducting Half Plane

In the previous section we considered some electromagnetic problems arising from source distributions having a special angular dependence. However our basic formulas (equations (9) and (9a) of Section II, to be denoted by (2.9) and (2.9a)) can also be used to good purpose in deducing the effect of a conducting half-plane on general incident fields, as we shall see below.*

We consider the problem of diffraction of an arbitrary vector electromagnetic field by a perfectly conducting half-plane. The half-plane is defined by $Y = 0$, $X > 0$ as in Fig. 1(a). We use two sets of coordinates, the Cartesian coordinates X, Y, Z, and the cylindrical coordinates ρ, ϕ, Z, defined in (2.9):

$$X = \rho \cos \phi, \quad Y = \rho \sin \phi; \quad 0 \leq \phi \leq 2\pi . \tag{1}$$

The total electric field $\underline{\mathcal{E}}$ is expressed by means of its Cartesian components $\mathcal{E}_X$, $\mathcal{E}_Y$, $\mathcal{E}_Z$, which must obey the inhomogeneous wave equations

$$(\nabla^2 + k^2)\mathcal{E}_X = S_X; \; (\nabla^2 + k^2)\mathcal{E}_Y = S_Y; \; (\nabla^2 + k^2)\mathcal{E}_Z = S_Z , \tag{2}$$

the divergence condition

$$\nabla \cdot \underline{\mathcal{E}} = \frac{\partial \mathcal{E}_X}{\partial X} + \frac{\partial \mathcal{E}_Y}{\partial Y} + \frac{\partial \mathcal{E}_Z}{\partial Z} = 0 \quad \text{outside of source regions,} \tag{3}$$

and the following boundary conditions on the perfectly conducting half-plane and at infinity:

$$0 = \mathcal{E}_X(X,0,Z) = \mathcal{E}_Z(X,0,Z) = \frac{\partial}{\partial Y}\mathcal{E}_Y(X,0,Z); \; X > 0. \; \left[\frac{\partial}{\partial Y}\mathcal{E}_Y(X,0,Z) \equiv \frac{\partial}{\partial Y}\mathcal{E}_Y\Big|_{Y=0}\right] \tag{4}$$

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* This problem was discussed by S. Karp in a paper[7] presented at the Symposium on Electromagnetic Waves held at the University of Michigan, July 1955

$$\mathcal{E}_X,\ \mathcal{E}_Y,\ \mathcal{E}_Z,\ \text{outgoing at infinity} \qquad (5)^*$$

Moreover, we require the edge conditions[4]

$$\mathcal{E}_Z(\rho,\ \phi,\ Z) \to 0 \quad \text{as} \quad \rho \to 0 \quad , \qquad (6)$$

no component of $\underline{\mathcal{E}}$ or $(\nabla\times\underline{\mathcal{E}})$ is as singular as $\frac{1}{\rho}$ at the edge . (7)

The functions S_X, S_Y, S_Z, in (2) represent the sources of the field, and are assumed to be given and to be confined to the interior of a finite region exterior to the half plane and its edge. Concerning (4) we note that the first two equalities express the boundary condition at a perfect conductor. The (redundant) condition $\frac{\partial \mathcal{E}_Y}{\partial Y} = 0$ follows from (3) if we proceed to the limit $Y \to 0$ for $X > 0$, while requiring that $\mathcal{E}_X$ and $\mathcal{E}_Z \to 0$. The magnetic field $\underline{\mathcal{H}}$ corresponding to $\underline{\mathcal{E}}$ according to the Maxwell field equations is <u>defined</u> as $\underline{\mathcal{H}} = (1/ik\eta)\ \nabla\times\underline{\mathcal{E}}$, where η is the characteristic impedance of free space. To facilitate subsequent discussions we introduce the incident field, $\underline{\mathcal{E}}_{inc}$. This field is defined as that outgoing wave solution of equation (2) which is regular except at the sources of the field. Since $\underline{\mathcal{E}}$ and $\underline{\mathcal{E}}_{inc}$ have the same sources we deduce from (2) that

$$(\nabla^2 + k^2)\ \nabla\cdot(\underline{\mathcal{E}} - \underline{\mathcal{E}}_{inc}) = \nabla\cdot\underline{S} - \nabla\cdot\underline{S} \equiv 0\ . \qquad (2a)$$

Now the function $\nabla\cdot(\underline{\mathcal{E}} - \underline{\mathcal{E}}_{inc})$ is an outgoing wave at infinity, as follows from the outgoing behavior of the Cartesian components of $\underline{\mathcal{E}}$ and $\underline{\mathcal{E}}_{inc}$. Since it has no sources (by (2a)), it vanishes identically. Thus (3) can be replaced by:

$$\nabla\cdot\underline{\mathcal{E}} = \nabla\cdot\underline{\mathcal{E}}_{inc} = 0 \quad \text{outside of sources} \qquad (3a)$$

$$\nabla\cdot\underline{\mathcal{E}} - \nabla\cdot\underline{\mathcal{E}}_{inc} \equiv 0 \quad \text{everywhere}\ .$$

---

* If $k = k_1 + ik_2$, $k_2 > 0$, then we require instead that $\underline{\mathcal{E}}(\infty) = 0$.

Our method of analysis will be based on an assumed knowledge of the solutions of certain scalar boundary value problems in terms of which the electromagnetic solution will be expressed. In the scalar diffraction theory for a half-plane it is customary to solve problems of the type posed in (2.2) and (2.3) and summarized below for convenience:

$$\nabla^2 \phi + k^2 \phi = \text{source term} \tag{8}$$

$$\phi \text{ outgoing at } \infty \tag{9}$$

$$\phi(X,0,Z) = 0, \text{ or } \frac{\partial}{\partial Y} \phi(X,0,Z) = 0, \text{ for } X > 0. \tag{10}$$

$$\phi \text{ finite as } \rho \rightarrow 0 \tag{11}$$

The pair of problems so expressed can be solved explicitly, as is well known. We can, therefore, construct directly a triplet of functions (E_X, E_Y, E_Z), i.e., a vector $\underline{E}$, say, which satisfies conditions (2), (4) and (5), and such that $\underline{E}$ is finite at the edge of the screen. Despite this, the vector $\underline{E}$ fails to be a solution of Maxwell's equations, since it does not satisfy the divergence condition (3).

It is known that near the edge, functions of the type of (E_X, E_Y, E_Z) behave like

$$(C_1(z)\sqrt{\rho} \sin \phi/2,\ C_2(z) + C_3(z)\sqrt{\rho} \cos \phi/2,\ C_4(z)\sqrt{\rho} \sin \phi/2) \tag{12}$$

where the functions C_1, C_2, C_3 and C_4 depend on the particular excitation. Hence conditions (6) and (7) are actually fulfilled. In fact they are over-fulfilled, since we can allow appropriate infinities of $\mathcal{E}_X$ and $\mathcal{E}_Y$ at the edge.

We now proceed to show how to construct the vector $\underline{\mathcal{E}}$, given the vector $\underline{E}$. To do this we define $\underline{e} = \underline{\mathcal{E}} - \underline{E}$, and then note that we must have, in virtue of (2)-(7) and (3a),

$$(\nabla^2 + k^2)\, \underline{e} = 0 \; , \quad \underline{e} \equiv (e_X, e_Y, e_Z) \; , \tag{13}$$

$$\psi \equiv \nabla\cdot\underline{e} + \nabla\cdot\underline{E} - \nabla\cdot\underline{\mathcal{E}}_{inc} = 0 \; , \tag{14}$$

$$e_X(X,0,Z) = e_Z(X,0,Z) = \frac{\partial}{\partial Y}\, e_Y(X,0,Z) = 0; \; X > 0 \; , \tag{15}$$

$$\underline{e} \text{ outgoing at } \infty \; , \tag{16}$$

$$e_Z(\rho, \phi, Z) \to 0 \quad \text{as} \quad \rho \to 0 \tag{17}$$

$$\underline{e}, \; (\nabla\times\underline{e}) \text{ not as singular as } \frac{1}{\rho} \text{ at } \rho = 0 \; . \tag{18}$$

If such a vector $\underline{e}$ can be found, then $\underline{e} + \underline{E}$ will satisfy all the conditions imposed upon $\underline{\mathcal{E}}$.

It is at this point that the basic formulas (2.9) and (2.9a) are employed. Let us set

$$e_X = \frac{1}{\sqrt{\rho}} \sin \phi/2 \;\; F_1(\rho, Z) \; , \tag{19}$$

$$e_Y = \frac{1}{\sqrt{\rho}} \cos \phi/2 \;\; F_2(\rho, Z) \tag{20}$$

$$e_Z \equiv 0 \; , \tag{21}$$

where

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{\partial^2}{\partial Z^2} + k^2\right) \; F_{1,2} = 0 \; , \tag{22}$$

and where F_1 and F_2 are outgoing at infinity. Then (13), (15), (16) and (17)

are fulfilled. We can exclude the possibility that $F_1(0, Z) = 0$, or $F_2(0, Z) = 0$, for then the functions F_1 and F_2 would be identically zero, since they are sourceless radiating wave function in the Cartesian ρ, Z plane. Thus $e_X \to \rho^{-1/2} \sin(\phi/2)F_1(0, Z)$, $e_Y \to \rho^{-1/2} \cos(\phi/2)F_2(0, Z)$ near $\rho = 0$, and this is the expected form of edge singularity, and it is in accordance with (18), insofar as $\underline{e}$ is concerned. Condition (18) also restrict the singularity of $(\nabla\times\underline{e})$, of course, but this will be attended to after the solution is obtained.

We now have to fulfill condition (14). To this end we note that the function $\psi = \nabla\cdot\underline{e} + \nabla\cdot\underline{E} - \nabla\cdot\underline{\mathcal{E}}_{inc}$, with $\underline{e}$ given in (19)-(21), is a scalar wave function which is outgoing at infinity. This follows from the fact that the Cartesian components of the vector $(\underline{e} + \underline{E} - \underline{\mathcal{E}}_{inc})$, and therefore their derivatives as well, possess these properties. Furthermore, we know that ψ vanishes on the screen. [In fact $\nabla\cdot\underline{e} = 0$ on the screen by (15), $\nabla\cdot\underline{E}$ vanishes there because of (4), while $\nabla\cdot\underline{\mathcal{E}}_{inc} = 0$ at the screen since the sources do not extend to the screen.] These properties of ψ ensure that it will vanish identically provided that it vanishes as the edge is approached radially. We shall therefore examine the behavior of ψ near the edge; as we shall see, the divergence will vanish there provided the functions F_1 and F_2 are suitably chosen.

First, as mentioned above, $\nabla\cdot\underline{\mathcal{E}}_{inc} = 0$ near the edge. Next we observe from (12), that the singular part of the expansion of $\nabla\cdot\underline{E}$ is given by the formula (note: $\frac{\partial}{\partial X}(\sqrt{\rho}\,\sin\,\phi/2) = -\frac{\partial}{\partial Y}(\sqrt{\rho}\,\cos\,\phi/2) = -\frac{1}{2\sqrt{\rho}}\sin\,\phi/2)$

$$\nabla\cdot\underline{E} \to \frac{-1}{2}\, C_1(Z)\,\frac{1}{\sqrt{\rho}}\,\sin\,\phi/2 + \frac{1}{2}\, C_3(Z)\,\frac{1}{\sqrt{\rho}}\,\sin\,\phi/2 = \frac{C(Z)}{\sqrt{\rho}}\,\sin\,\phi/2 \qquad (23)$$

where

$$C(Z) = \frac{1}{2}\left[-C_1(Z) + C_3(Z)\right] \tag{23a}$$

Here the functions C_1, C_3 are known from the solution of the scalar problems. We therefore require that the singular part of the expansion of $\nabla\cdot\underline{e}$ be given by

$$\nabla\cdot\underline{e} \rightarrow -C(Z)\,\frac{1}{\sqrt{\rho}}\sin\phi/2\ , \tag{24}$$

so that $\nabla\cdot\underline{e} + \nabla\cdot\underline{E}$ will vanish as $\rho \rightarrow 0$. However, from (19), (20), (21) we have

$$\nabla\cdot\underline{e} \rightarrow \left[F_1(0,Z)\,\frac{\partial}{\partial X}\left(\frac{1}{\sqrt{\rho}}\sin\phi/2\right) + F_2(0,Z)\,\frac{\partial}{\partial Y}\left(\frac{1}{\sqrt{\rho}}\cos\phi/2\right)\right] \tag{25}$$

$$+ \frac{1}{\sqrt{\rho}}\sin\phi/2\cos\phi\,\frac{\partial}{\partial\rho}F_1(0,Z) + \frac{1}{\sqrt{\rho}}\cos\phi/2\,\frac{\partial}{\partial\rho}\,F_2(0,Z)\sin\phi$$

Since $F_1(0,Z)$ and $F_2(0,Z)$ do not both vanish, the bracketed term in (25) is $O(\rho^{-3/2})$; on the other hand the remaining part of the right side of (25) is $O(\rho^{-1/2})$. Equation (24) therefore implies that the bracketed term of (25) vanishes identically. But the coefficients of F_1 and F_2 in (25) are identical by the Cauchy-Rieman equations, since $(X + iY)^{-1/2} = \frac{1}{\sqrt{\rho}}(\cos\phi/2 - i\sin\phi/2)$.

Hence we conclude that

$$F_1(0,Z) + F_2(0,Z) = 0 \tag{26}$$

The uniqueness theorem for two dimensional scalar waves then implies that $F_1(\rho,Z) + F_2(\rho,Z) \equiv 0$ for all ρ, since the functions F_1 and F_2 are sourceless and outgoing. Therefore also $\frac{\partial}{\partial\rho}F_1(0,Z) = -\frac{\partial}{\partial\rho}F_2(0,Z)$. We now compare the

second part of the right side of (25) with the right side of (24), and use the identity of $\frac{\partial F_1}{\partial \rho}(0,Z)$ and $\frac{-\partial F_2}{\partial \rho}(0,Z)$ to find

$$-C(Z)\sin \phi/2 = \frac{\partial}{\partial \rho} F_2(0,Z)\left[\sin \phi \cos \phi/2 - \cos \phi \sin \phi/2\right] \tag{27}$$

$$= \frac{\partial}{\partial \rho} F_2(0,Z) \sin \phi/2 \quad .$$

Hence we have to determine $F_2(\rho,Z) = - F_1(\rho,Z)$ from the condition that

$$\frac{\partial}{\partial \rho} F_2(0,Z) = -C(Z) \quad . \tag{28}$$

But $F_2(\rho,Z)$ is an outgoing wave function of ρ and $Z(=z)$ if these are regarded as Cartesian coordinates. Hence we can solve (28) with (22) by a direct application of Green's theorem to the half space region $\rho \geq 0$, utilizing the two-dimensional Neumann type half-space Green's function, as follows

$$F_2(\rho,Z) = -\frac{i}{2}\int_{-\infty}^{\infty} C(z_o)\, H_o^{(1)}\left(k\sqrt{\rho^2 + (Z-z_o)^2}\right) dz_o = - F_1(\rho,Z) \tag{29}$$

We can now insert (29) into (19) and (20), and this gives us the solution for the required vector $\underline{e}$. Then $\underline{\mathcal{E}} = \underline{E} + \underline{e}$, where $\underline{E}$ is composed of the familiar scalar solutions, and $\underline{e}$ is given by quadratures in terms of the quantity

$$C(Z) = \lim_{\rho \to 0} \frac{\sqrt{\rho}}{\sin \phi/2} \nabla \cdot \underline{E} \quad . \tag{30}$$

If we use the fact that $F_1(\rho,Z) = - F_2(\rho,Z)$ it is easy to verify from (19), (20) and (21) that $\nabla \times \underline{e}$ is $O(\rho^{-1/2})$ at the edge, as required by (18). The analysis is therefore complete.

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